2001-2002 **American Computer Science League** **Contest #3**

Senior Division **Solutions**

1. Digital Electronics

The circuit translates as follows: $A\overline{B} + C$. This simplifies to: $(\overline{A\overline{B}})\ \overline{C} = (\overline{A} + \overline{\overline{B}})\ \overline{C} = (\overline{A} + B)\ \overline{C}$. To be TRUE, both factors must be TRUE. This implies that C must equal 0. The first factor gives 2 choices. If A = 0 then B can be either 1 or 0. If A = 1 then B must equal 1. The choices are (*, 1, 0) and (0, 0, 0).

1. (*, 1, 0) and (0, 0, 0).

OR

(0,1,0), (1,1,0), and (0, 0, 0).

2. Digital Electronics

The circuit is equivalent to: $(A + B)((B \oplus C)\ D)$.

Substituting 0 for B gives : $(A + 0)((0 \oplus C)\ D) = (\overline{A})\ (CD)$

2 $\dot{\overline{A}}\,CD$

3. Boolean Algebra

$(A+B) \oplus (AB) = (A+B)\ (A\ B) + (A + B)\overline{(AB)} =$
$\overline{A}\ \overline{B}\ AB + (A + B)\ (\overline{A} + \overline{B}) = 0 + A\overline{A} + \overline{A}\ B + A\overline{B} + B\overline{B} =$
$0 + 0 + A \oplus B + 0 = A \oplus B$

3. $A \oplus B$

4. Graph Theory

The adjacency matrix is constructed as follows:

	A	B	C	D	E	F
A	0	1	1	1	0	0
B	1	0	0	0	0	1
C	1	0	0	0	0	0
D	0	0	1	0	1	0
E	1	0	0	0	0	1
F	0	0	0	0	1	0

4. 11

5. Graph Theory

Number the vertices 1 through 10. Start with vertex 1 and draw the edges to vertices 2 through 10. There are 9. Next draw the edges from vertex 2 to vertices 3 through 10. There are 8. Repeating the process and summing the number of edges (9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45).

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