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⋆ 1996–97 All-Star Contest ⋆ Short Problems ⋆ Round #1 ⋆

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1. [Jr] BASIC Expressions

Write a BASIC expression that is an exact translation of the following:

> *Twice the difference of the product of A and B and the quotient of C and D*

Use the fewest number of parentheses as possible.

2. [Jr, Int, Sr] Boolean Algebra

List all of the following choices that are equivalent to the expression

$$(A \oplus B)\overline{(\overline{A} + B)(A + \overline{B})}$$

(**a**) $\overline{A} \oplus B$
(**b**) $\overline{A} + \overline{B}$
(**c**) $\overline{A}B$
(**d**) $A + B$
(**e**) $A \oplus B$

3. [Int, Sr] What Does This Program Do?

If the following program prints the number 9 when it is run, list all possible integer inputs to the program.

```
10 input A : B=3 : C=1
20 if (A<=B) and ((C<A) or (B<C)) then A = A+2
30 if (A<B) and (B>C) then A = A+C
40 if ((A>C) and (A>B)) or (B>C) then A = A-C
50 if A>5 then A=A+2 else A=A+5
60 print A
70 end
```

4. [Int, Sr] Prefix/Postfix/Infix Notation

Let # represent the unary operation "greatest integer less than or equal to the value of its operand." For example, the postfix expression

```
26 3 / #
```

has a value of 8. Evaluate the following prefix expression:

```
+ # / * 2 3 4 / # - 1 3 # / - ↑ 6 2 4 6
```

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1. The answer shown is the only correct answer.

`2*(A*B-C/D)`

2. The expression is simplified by applying DeMorgan's Law twice and then the definition of XOR:

$$\begin{aligned}(A \oplus B)\overline{(\overline{A}+B)(A+\overline{B})} &= (A \oplus B)(\overline{(\overline{A}+B)} + \overline{(A+\overline{B})}) \\ &= (A \oplus B)((\overline{\overline{A}} \cdot \overline{B}) + (\overline{A} \cdot \overline{\overline{B}})) \\ &= (A \oplus B)((A\overline{B}) + (\overline{A}B)) \\ &= (A \oplus B)(A \oplus B) \\ &= (A \oplus B)\end{aligned}$$

The only choice equivalent to $A \oplus B$ is choice (**e**).

(**e**)

3. We can solve this program by trying small values, say between -10 and 10, looking for inputs that cause the program to print 9. If you follow this approach, you need to be able to convince yourself that all values less than -10 and greater than 10 won't work. Alternatively, work backwards, one line at a time:

Line 50: After this line is executed, `A` must have a value of 9. If the `then` were executed, `A` must have started out equal to 7. If the `else` were executed, `A` must have started out with a value of 4.

Line 40: After this line is executed, `A` must have a value of 4 or 7. The `then` is always executed because `B>C`. Thus, `A` must have started out with a value of 5 or 8.

Line 30: After this line is executed, `A` must have a value of 5 or 8. If the `then` were executed, then `A` must have started out equal to 4 or 7. But these are both impossible, because `A` must be less than `B`. Therefore, the `then` was not executed, and `A` started out with a value of 5 or 8.

Line 20: After this line is executed, `A` must have a value of 5 or 8. Consider a final value of 8. If the `then` were executed, `A` started out with a value of 6. But this is impossible, since `A` must be less than `B`; thus `A` started out equal to 8. Consider a final value of 5. If the `then` were executed, then `A` started out with a value of 3. This is OK. If the `then` were not executed, then `A` start out with a value of 5. This is also OK.

3, 5, and 8

4. The evaluation is as follows (a box is put around the part of the expression that is about to be evaluated):

```
+ # / [* 2 3] 4 / # [- 1 3] # / - [↑6 2] 4 6
+ # [/ 6 4] / [# -2] # / [- 36 4] 6
+ [# 1.5] / -2 # [/ 32 6]
+ 1 / -2 [# 5.333...]
+ 1 [/ -2 5]
[+ 1 -.4]
.6
```

.6

the fraction 3/5 is also acceptable

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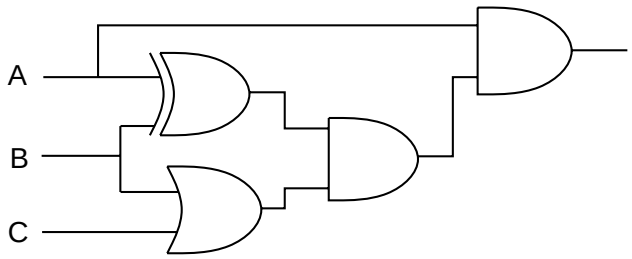
5. [Jr] What Does This Program Do?

When the following program is run, the string "hi mom!" is printed three times. Find all integer values that could be given as input.

```
10 input x : y = 7
20 if (x < 2) and (y > x) then print "hi mom!"
30 if (x > y) or (x < 5) then print "hi mom!"
40 if (x <> 0) and (x*x < 5) then print "hi mom!"
50 end
```

6. [Jr, Int, Sr] Digital Electronics

Find all ordered triples that make the circuit below true.



7. [Int, Sr] Pascal

After the following program is run, what is the final value of `a`?

```
program AllStar97;
  var a, b, c: integer;
  procedure two (var c, b, a: integer);
    begin
      a:=a-2;
      b:=b-1;
      c:=a+b;
    end;
  procedure one (var a, b, c: integer);
    begin
      a:=a-1;
      b:=b-1;
      c:=a+b;
      two(a,b,c);
    end;
  begin
    a:=20; b:=10; c:=0;
    while a > 0 do begin
      one(a,b,c);
      c:=c+1;
    end;
  end.
```

8. [Int, Sr] Lisp

Evaluate the following expression:

```
(CAR (REVERSE (CDR (CDR '((1 2) (3 4) (5 6 7))))))
```

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5. In order for the string to be printed three times, it must be printed in lines 20, 30, and 40. From line 20, `x` $= 1, 0, -1, -2, \ldots$ will cause the string to be printed. All of these values will cause line 30 to print the string. In line 40, both clauses of the `if` must be true for the string to be printed. The clause `x<>0` eliminates 0 from the possible values of `x`. The clause `x*x<5` is true only when `x`=1, −1, and −2.

1, −1, and −2

6. The circuit can be represented by the Boolean Algebra expression

$$A((A \oplus B)(B + C))$$

We could construct a truth table for this expression, but because it's an AND of three terms, it's worth trying to reason about it. The term A means that A must be true. This gives us 4 possible triples to consider: (1,0,0), (1,0,1), (1,1,0), and (1,1,1). The term $A \oplus B$ means that A and B must be different. This leaves just (1,0,0) and (1,0,1). The term $B + C$ means than either B or C must be true. This leaves just (1,0,1).

(1,0,1)

7. The table at the right shows the values of variables just after `c` is incremented in the main loop.

a	b	c
34	8	27
44	6	39
50	4	47
52	2	51
50	0	51
44	−2	47
34	−4	39
20	−6	27
2	−8	11
−20	−10	−9

−20

8. The evaluation goes from the inside to the outside:

```
(CDR '((1 2) (3 4) (5 6 7)))  ⇒  ((3 4)(5 6 7))
      (CDR '((3 4) (5 6 7)))  ⇒  ((5 6 7))
        (REVERSE '((5 6 7)))  ⇒  ((5 6 7))
            (CAR '((5 6 7)))  ⇒  (5 6 7)
```

(5 6 7)

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9. [Jr, Int] Computer Number Systems

Express the square of 878_{16} in base 16.

10. [Jr, Int, Sr] Bit String Flicking

Find all values of `x`, 5 bits long, that make the following expression equal to 10100.

```
(LSHIFT-2 (10101 OR x)) XOR (RCIRC-2 (11011 AND (LCIRC-2 x)))
```

11. [Int, Sr] Recursive Functions

Evaluate $f(22)$ given the following:

$$f(x) = \begin{cases} x + f(x+3) & \text{whenever } x \text{ is prime and larger than 2} \\ \frac{x}{2} * f(x-5) & \text{whenever } x \text{ is even} \\ x+1 & \text{otherwise} \end{cases}$$

12. [Sr] Data Structures

If you built a binary search tree with the keys S A L T L A K E (in this order), you'd end up with a tree whose internal path length is 18. Suppose that you could swap up to 2 pairs of letters (e.g., the S with the first L and the T with the K) to insert the letters in a different order (e.g., L A S K L A T E, using the 2 swaps above). What is the minimum internal path length a resulting tree could have?

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9. We could convert the number to base 10, square it, and then convert back to base 16. Alternatively, we could do all the multiplication in base 16. We'll do the latter, but take advantage of the fact that

$$(a+b)^2 = a^2 + 2ab + b^2$$

to make the multiplication simple.

$$\begin{aligned} 878^2 = (800+78)^2 &= 800^2 + 2\cdot 800\cdot 78 + 78^2 \\ &= 400000 + 1000\cdot 78 + 78^2 \\ &= 400000 + 78000 + 78^2 = 478000 + 78^2 \end{aligned}$$

$$\begin{aligned} 78^2 = (70+8)^2 &= 70^2 + 2\cdot 70\cdot 8 + 8^2 \\ &= 3100 + 700 + 40 = 3840 \end{aligned}$$

Finally, $478000 + 3840 = 47\text{B}840$.

$47\text{B}840_{16}$

10. Write `x` as `abcde` and simplify each side:

Left side of XOR: `(LSHIFT-2 (10101 OR abcde))`
`(LSHIFT-2 1b1d1)`
`1d100`

Right side of XOR: `(RCIRC-2 (11011 AND (LCIRC-2 abcde)))`
`(RCIRC-2 (11011 AND cdeab))`
`(RCIRC-2 cd0ab)`
`abcd0`

Now, these two values XOR'd together must equal 10100. That is, `1d100 XOR abcd0 = 10100`. Look at each bit, from left-to-right:

`1 XOR a = 1` implies `a=0`
`d XOR b = 0` implies `b=d`
`1 XOR c = 1` implies `c=0`
`0 XOR d = 0` implies `d=0`, and thus `b=0`

00000 and 00001

the term 0000 is also acceptable*

11. The evaluation is as follows:

$$\begin{aligned} f(22) &= 22/2 * f(17) = 11 * f(17) \\ f(17) &= 17 + f(20) \\ f(20) &= 20/2 * f(15) = 10 * f(15) \\ f(15) &= 16 \end{aligned}$$

Working backwards, we have $f(20) = 10*16 = 160$, $f(17) = 17+160 = 177$, and $f(22) = 11 * 177 = 1947$.

1947

12. The shortest internal path length occurs when the binary tree is a complete tree. With 8 nodes, the shortest internal path length is 13. Such a tree can be formed with just one swap: the K and the S. (Other exchanges will also produce a tree whose internal path length is 13.)

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13. [Int, Sr] Assembly Language

What is the final value of `C` when the following program is run?

```
A    DC    2
B    DC    50
C    DC    0
TOP  LOAD  C
     ADD   =1
     STORE C
     LOAD  B
     SUB   A
     STORE B
     SUB   C
     BG    TOP
     END
```

14. [Int, Sr] FSAs and Regular Expressions

How many different strings of length 5 are generated by the following regular expression:

`(WW+A)*CS*L`

15. [Int, Sr] Data Structures

Insert the nodes S A L T L A K E into a *minimal heap*. That is, a heap where each node is less than or equal to its two children. (This is different than the heap described in the ACSL Category Description booklet, where nodes were greater than their children.) Draw the resulting heap.

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13. The high-level program equivalent to the program is as follows:

```
      A = 2
      B = 50
      C = 0
TOP:  C = C + 1
      B = B - A
      IF B-C > 0 THEN GOTO TOP
      END
```

The following table shows the values of `B` and `C` just before the `BG` is executed:

B	C
48	1
46	2
44	3
...	...
20	15
18	16
16	17

17

14. All strings must end with an `L` and must contain a `C`. Between the `C` and `L`, there are zero or more `S`'s. Before the `C`, there's either nothing, or some combination of `A` and `WW`, for instance `AWW`, `AA`, or `WWA`. The complete list of 5-letter strings are as follows: `CSSSL`, `ACSSL`, `AACSL`, `AAACL`, `WWCSL`, `WWACL`, and `AWWCL`.

7

15. Here's what the tree looks like after each letter is added: